MAT 1152 GENEL MATEMATİK II CEVAP ANAHTARI

1) a) $\int x \arctan x \, dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\,dx$

$\ulcorner$ $u = \arctan x$, $dv = x\,dx$

$du = \frac{dx}{1+x^2}$, $v = \frac{x^2}{2}$ $\lrcorner$ $\quad = \frac{x^2}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + c$

b) $\int \frac{x^2\,dx}{\sqrt{16-x^2}} = (Ax+B)\sqrt{16-x^2} + \lambda\int\frac{dx}{\sqrt{16-x^2}}$

$\frac{x^2}{\sqrt{16-x^2}} = A.\sqrt{16-x^2} - \frac{x}{\sqrt{16-x^2}}(Ax+B) + \frac{\lambda}{\sqrt{16-x^2}}$

$x^2 = 16A - Ax^2 - Ax^2 - Bx + \lambda$

$x^2 = -2Ax^2 - Bx + 16 + \lambda \Rightarrow A = -1/2$, $B = 0$, $\lambda = 8$

$\int\frac{x^2\,dx}{\sqrt{16-x^2}} = -\frac{1}{2}x\sqrt{16-x^2} + 8\int\frac{dx}{\sqrt{16-x^2}} = -\frac{1}{2}x\sqrt{16-x^2} - 8\arcsin\frac{x}{4} + c$

c) $\int\frac{dx}{5+3\cos x} = \int\frac{2}{5+5t^2+3-3t^2}\,dt = \int\frac{2\,dt}{8+2t^2}$

$\ulcorner$ $\tan\frac{x}{2} = t$, $dx = \frac{2}{1+t^2}dt$ $\quad = \int\frac{dt}{4+t^2} = \frac{1}{2}\arctan\frac{t}{2} + c$

$\cos x = \frac{1-t^2}{1+t^2}$ $\lrcorner$

$= \frac{1}{2}\arctan\left(\frac{\tan x/2}{2}\right) + c$

d) $\int\frac{2x-1}{x(x^2+1)}\,dx = \int\frac{A}{x}\,dx + \int\frac{Bx+C}{x^2+1}\,dx = -\ln|x| + \frac{1}{2}\ln|x^2+1| + 2\arctan x + c$

$\ulcorner$ $(A+B)x^2 + Cx + A = 2x - 1$, $C = 2$, $A = -1$, $B = 1$ $\lrcorner$

e) $\int x 2^{-x}\,dx = -\frac{x2^{-x}}{\ln 2} + \int\frac{2^{-x}}{\ln 2}\,dx$

$\ulcorner$ $x = u$, $2^{-x}dx = dv$

$dx = du$, $-\frac{2^{-x}}{\ln 2} = v$ $\lrcorner$ $\quad = -\frac{x2^{-x}}{\ln 2} + \left(-\frac{2^{-x}}{(\ln 2)^2}\right) + c$